

Calculating Radius of Turn from Yaw Rate



Abstract

A number of vehicle tests require or make use of an accurate radius of turn measurement. Currently Racelogic VBOX Tools offers a radius of turn channel generated from the GPS signal that can be calculated by any VBOX. However this is a relatively noisy signal, and is only true for the radius of turn at the reference antenna location (typically the rear of the vehicle). Below shows an example radius of turn (blue) calculated from the GPS signal that highlights how noisy a signal can be, even at 33km/h.

This noisy trace is a particular problem in tests such as test 6.3.2 of ECE 79 where a vehicle and trailer are required to make a steady state 25m radius turn at 5km/h. At these speeds the heading is particularly noisy and so it is advised to use an IMU to calculate radius of turn from yaw rate.

There is also a growing requirement to show the radius of turn on various point of the body, for example in heavy vehicle tests to show that the tail swing is not hugely different from the front of the vehicle. By using a dual antenna VBOX 3iSL in combination with an IMU it is also possible to mathematically translate the radius of turn measurement to any point on the vehicle, allowing the user to calculate the difference in radii between points on the body.

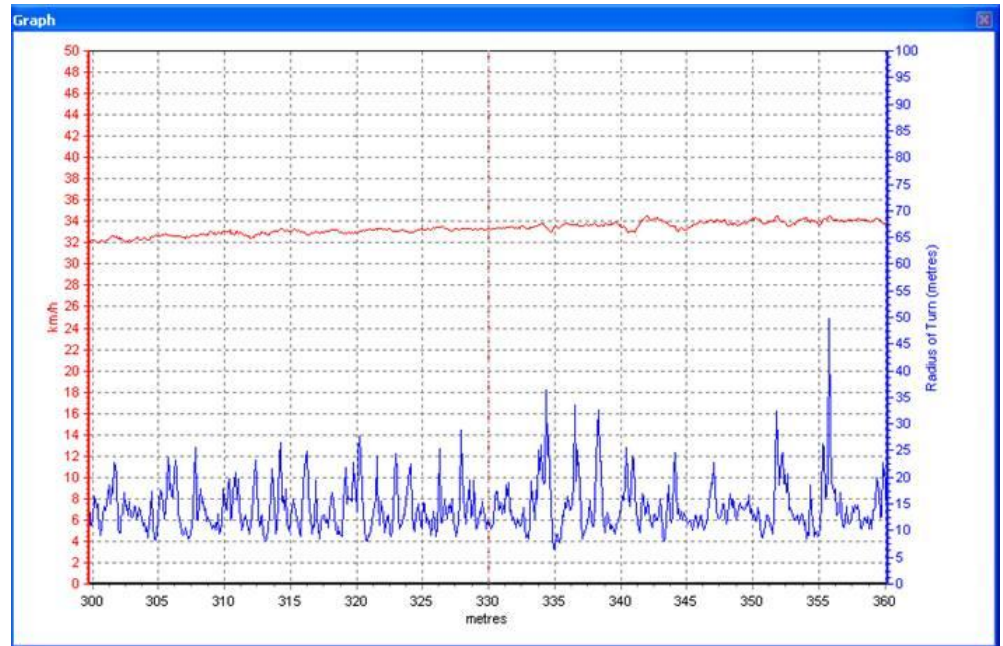


Figure 1: Graph that shows speed (red) and Radius of Turn calculated via GPS (blue)

This document demonstrates the work undertaken by Racelogic to determine the radius of turn from yaw rate and also the radius of turn at the front of a vehicle, away from any antenna mounting locations.

Equipment Required

To calculate the radius of turn from yaw rate, an IMU can be used with any VBOX. However to translate the radius of turn to any point, a 100Hz VBOX 3iSL is used in conjunction with an IMU to obtain precise values for heading, true heading and yaw rate, something which is not possible using a standalone VBOX. All of this is controlled and calibrated using the VBOX Manager meaning the tests can be conducted without the need for an in-car PC.

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Installation and Configuration

Hardware

To calculate an accurate radius of turn only, it is advised to mount the GPS antenna on the point where the radius of turn is required to be measured. The IMU can then be placed anywhere on the vehicle (as long as it is a rigid, non-articulated vehicle) as solid bodies exhibit the same yaw rate throughout. If the user wants to translate the radius of turn, then they will have to use the 100Hz VBOX 3iSL module with the twin antenna configuration set out on top of the vehicle as shown below:



Figure 2: Twin antenna configuration

It is important that the distance between the two antennas is recorded and used in the VBOX Manager to calibrate the system and reduce error. Please refer to VBOX 3iSL 100Hz Data Logger User Guide for further calibration information.

When a vehicle is making a constant turn the relationship between yaw rate and velocity is constant and with this information the radius of turn can be calculated using the following equation:

$$\text{Radius of turn} = \frac{\text{Velocity}}{\text{Yaw Rate}}$$

By using the IMU an enhanced yaw rate channel is recorded within the VBOX along with the built in speed channel a highly accurate and reliable radius of turn value is generated. This can be seen below in the same graph as shown in figure 1, only with the addition of the new, less noisy and more accurate calculated radius of turn (yellow)

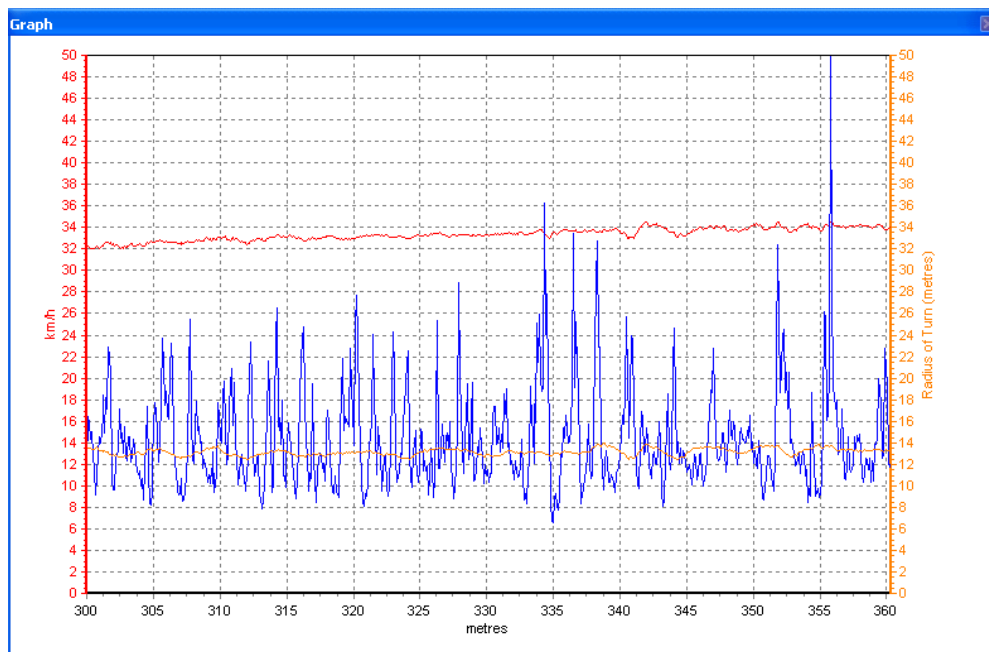


Figure 3: Graph to show the difference between the Radius of turn calculated by GPS (blue) compared to radius of turn calculated using Yaw Rate (orange)

When using a twin antenna system, the velocity used to calculate the radius of turn is taken from the reference antenna mounted at the rear of the vehicle; the radius of turn value is therefore only valid at the rear of the vehicle.

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The diagram on the right explains how the radius of turn can now be derived for the front left of the vehicle using the radius of turn value calculated above and the slip angle of the vehicle. The two red points are the location of the VBOX 3iSL antennas, whilst the black point is the point of interest.

To calculate the length of 'a' the standard cosine rule can be applied:

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$

In this application a = radius of turn at the Front Left of the vehicle, b = radius of turn at the rear of the vehicle, c = the distance between the reference antenna and the point of interest, and A = the angle between b and c. dx and dy are the lateral distances between the reference antenna and point of interest in the x and y plane.

This principle is not solely applicable to the front left of the vehicle, in-fact the same basic trigonometry can be applied to any point on the vehicle.

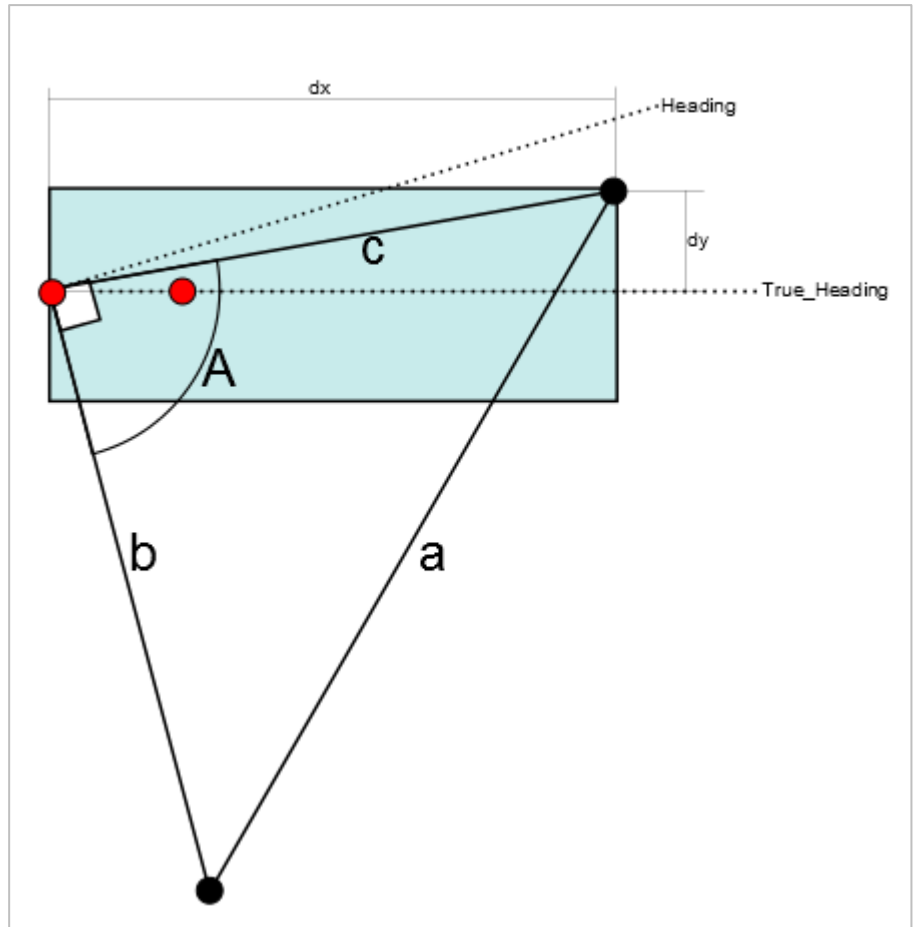


Figure 4: Radius of turn schematic

Software

Within VBOX Tools there are a number of maths channels that need implementing to develop the radius of turn. These equations are listed below:

Angle A	Radius b	Radius a
$90 - (YK - D) + \text{ARCTAN}(dy/dx)$	$((C * 0.2777) / (YQ * 0.0174))$	$\text{SQRT}((3.99^2) + (YY^2) - 2 * 3.99 * YY * \cos(YX))$
YK = True_Head D = Heading	C = Speed YQ = YawRate	3.99 = Distance c YY = Radius b YX = Angle a

Depending upon which channels that have been selected in the logger, the naming conventions may change within each equation.

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Testing Methodology

When a known reference radius is required, perform a number of test rotations at that radius and constant speed. Ensure all calibration tasks are completed regarding antenna offsets through the VBOX Manager.

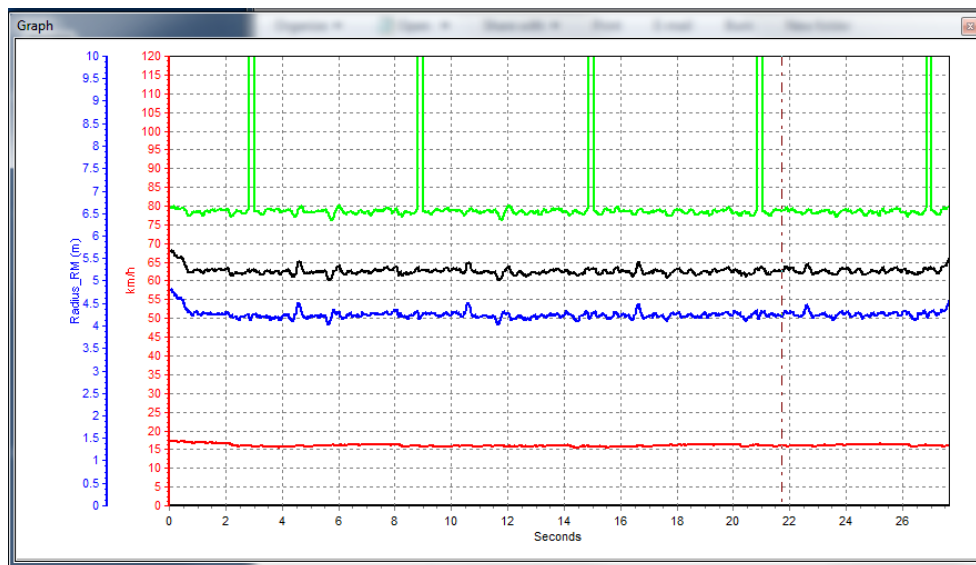
To use the maths channels detailed above the test must be performed with the vehicle traveling in a clockwise direction. If the test is performed in an anti-clockwise direction then the headings will need swapping on the angle calculation and the radius b value will need inverting (multiply by -1).

It is also important to use the correct signage with regards to the dy and dx values. For a right hand turn, the dy value should be POSITIVE when left of centre and NEGATIVE when right of centre, whereas in a left hand turn the opposite is true. dx values should always be the absolute values, i.e. always positive.

Sample Data and Interpretation

Once the data has been collected and downloaded into VBOX Tools the maths channels need executing. This will generate data similar to that shown below.

From the sample data the blue trace shows radius of turn measured at the rear of the car whilst the black trace is the calculated radius of turn at the front of the car.



Additionally the red trace shows the vehicle speed at the rear of the car and the green trace shows the calculated angle A.

Note that the value spikes when the True_Heading leads the Heading and crosses from 360° to 0°, this does not affect the accuracy of the results as when the cosine function is applied to the value it cancels the errors out.

Figure 5: Sample of radius of turn data

To validate the accuracy of the system we configured a car with a VBOX 3i in addition to the VBOX 3iSL. This was used to measure the velocity at the front of the vehicle and in turn generate a measured radius of turn at the front of the vehicle so we could compare the value to the calculated radius of turn.

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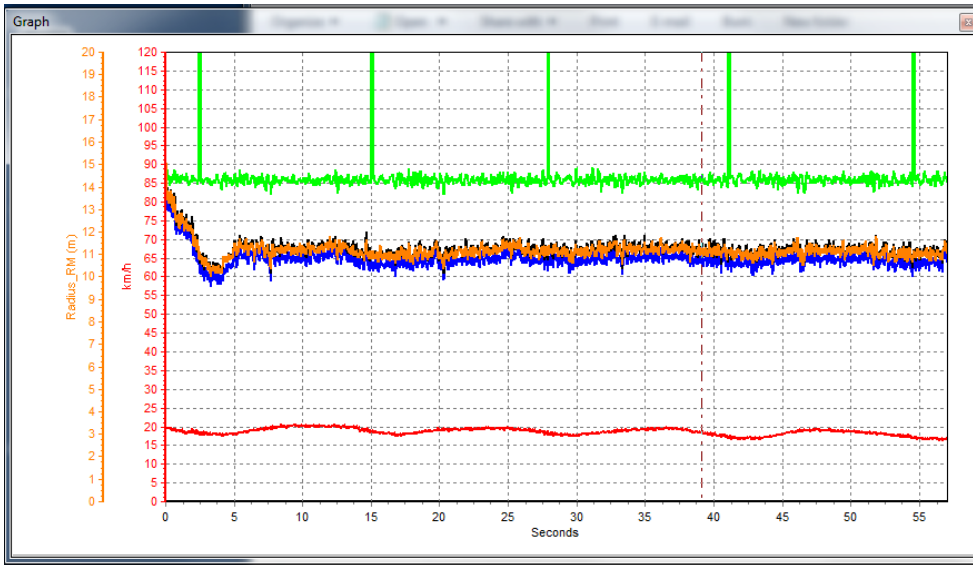


Figure 6: Sample of radius of turn data with a validation channel

This data shows results from the validation setup on a much larger radius of turn.

In this example the orange trace is the measured radius of turn at the front of the vehicle. Figure 5 shows an enhanced section of data which visually shows the similarities between the measured and calculated radius values.

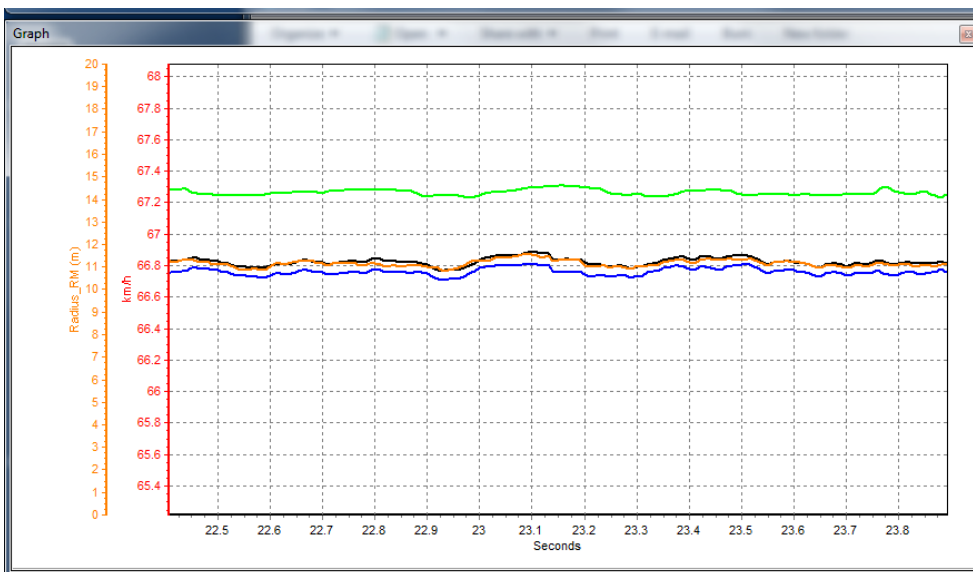


Figure 7: Enhanced data showing measured and calculated radius of turn

From this plot it is possible to see that the black and orange traces display a very small error characteristic. By using the measure function within VBOX Tools, the user can average the radius values over a selected period.

Below is the output table for the measure function on the data shown in figures 6 & 7. The values of most interest have been highlighted in yellow.

Channel	Start	End	Difference	(Max-b)	(Min-b)	Average
Time (s)	5.762	56.076	50.314	---	---	---
Speed (km/h)	18.700	16.960	-1.740	20.599	16.482	18.721
Distance (m)	30.008	291.216	261.208	---	---	---
Angle (°)	85.235	87.206	1.971	446.869	82.000	90.001
Radius_RM (m)	10.857	10.820	-0.037	11.579	9.840	10.764
Radius_FM (m)	11.170	11.110	-0.060	11.825	10.369	11.140
Radius_FC (m)	11.251	11.348	0.097	12.007	10.152	11.206

Table 1: Radius of turn sample data

They show that on average the measured radius of turn at the rear of the car is 10.764m, whilst the average measured radius of turn at the front of the car is 11.140m. Comparing the measured value to the calculated value of 11.206m shows a difference of only 0.06m, which over an 11.140m radius equates to less than 1% error.